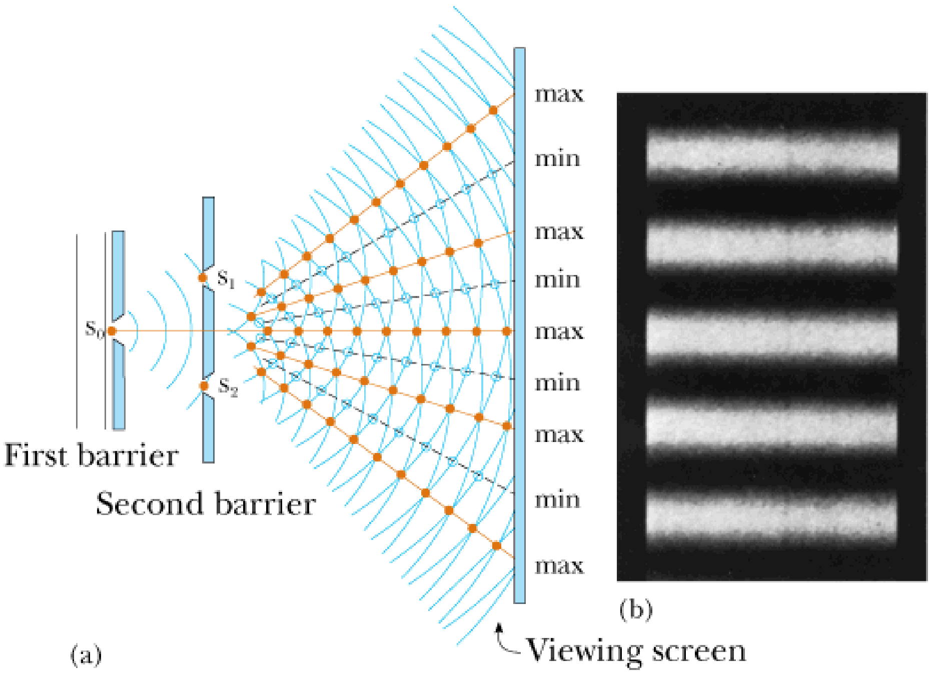
Interference of Light Waves

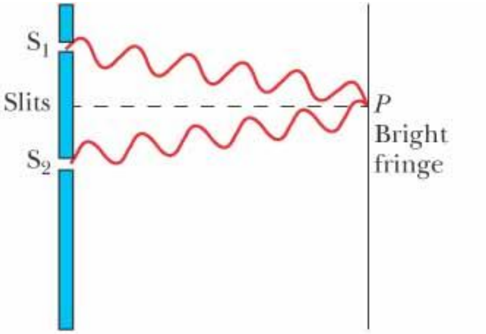
- Conditions for interference
- Young's double slit experiment
- Intensity distribution of the interference pattern
- Phasor representation
- Reflection and change of phase
- Interference in thin films

Conditions for Interference

- If two waves have a definite phase relationship then they are coherent.
- Otherwise, they are incoherent (ex: two light bulbs).
- For Interference:
  - The sources must be coherent.
  - The sources should be monochromatic.

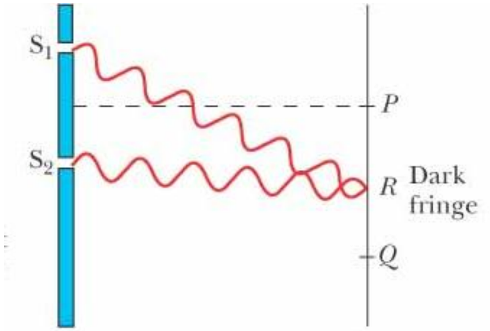
Young's Double-Slit Experiment





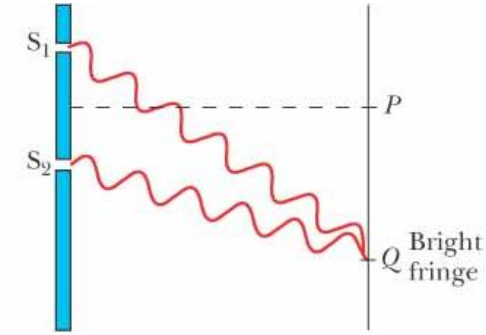
$\delta = 0$

$\Delta\phi = 0$



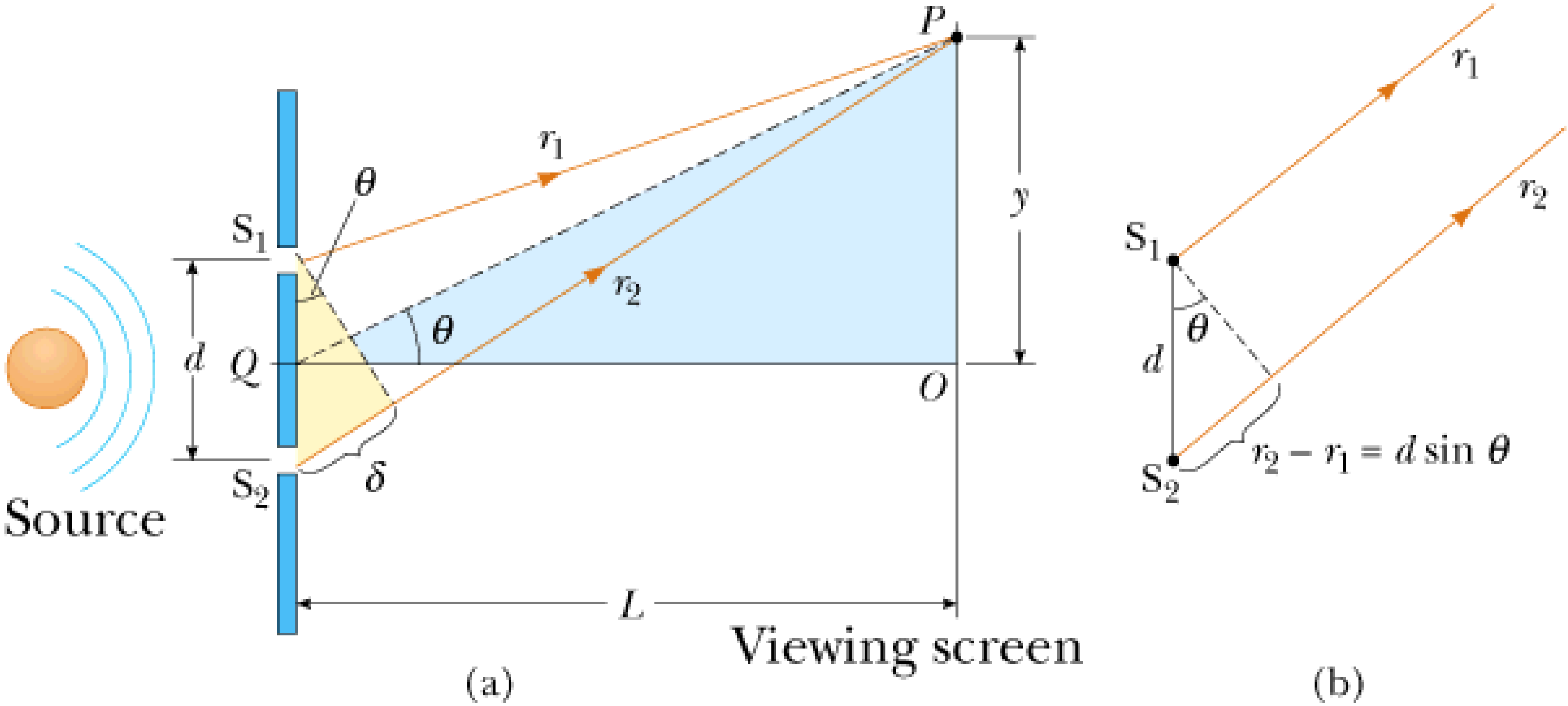
$\delta = \lambda/2$

$\Delta\phi = \pi$



$\delta = \lambda$

$\Delta\phi = 2\pi$



$$\delta = r_2 - r_1 = d\sin\theta$$

$$d >> \lambda \rightarrow \sin\theta \approx \tan\theta \rightarrow y = L\tan\theta \approx L\sin\theta$$

$$L >> d$$

$$\delta = d\sin\theta = m\lambda$$ Constructive interference

$$y_{bright} = \frac{\lambda L}{d} m$$

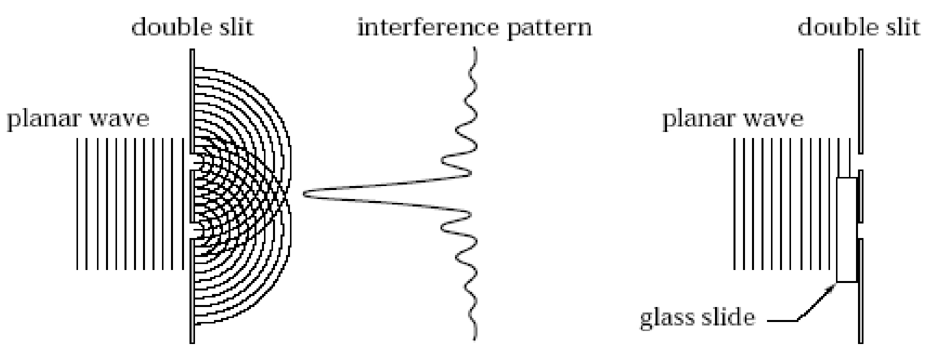
$$\delta = d\sin\theta = \left(m + \frac{1}{2}\right)\lambda$$ Destructive interference

$$m = 0, \pm 1, \pm 2, \ldots$$

$$y_{dark} = \frac{\lambda L}{d}\left(m + \frac{1}{2}\right)$$

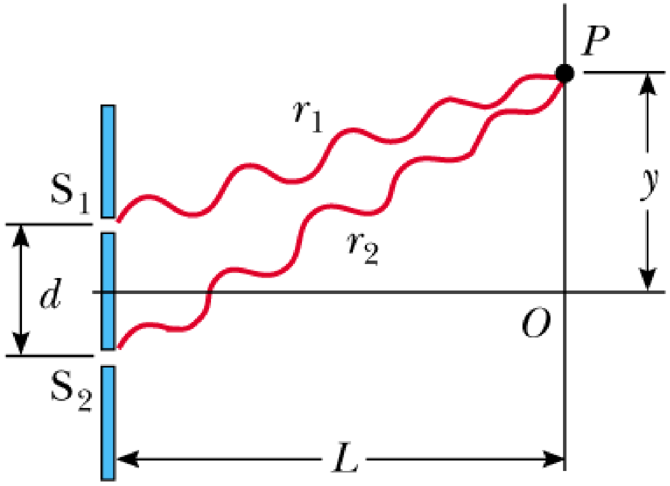
Concept Question

An interference pattern is formed on a screen by shining a planar wave on a double- slit arrangement (left). If we cover one slit with a glass plate (right), the phases of the two emerging waves will be different because the wavelength is shorter in glass than in air. If the phase difference is 180°, how is the interference pattern, shown left, altered?



1. The pattern vanishes.
2. The bright spots lie closer together.
3. The bright spots are farther apart.
4. There are no changes.
5. Bright and dark spots are interchanged.

Intensity Distribution of the Interference Pattern



$$E_1 = E_0 \sin \omega t \qquad E_2 = E_0 \sin(\omega t + \phi)$$

$$\delta = r_2 - r_1 = d \sin \theta$$

If $\delta = \lambda$ then $\phi = 2\pi$

$$\frac{\delta}{\lambda} = \frac{\phi}{2\pi} \longrightarrow \phi = 2\pi \frac{\delta}{\lambda} = \frac{2\pi}{\lambda} d \sin \theta$$

$$E_P = E_1 + E_2$$

$$E_P = E_0 [\sin \omega t + \sin(\omega t + \phi)]$$

$$E_P = 2E_0 \cos\left(\frac{\phi}{2}\right) \sin\left(\omega t + \frac{\phi}{2}\right)$$

$$I \propto E_P^{\ 2} = 4E_0^{\ 2} \cos^2\left(\frac{\phi}{2}\right) \sin^2\left(\omega t + \frac{\phi}{2}\right)$$

$$I = I_{\max} \cos^2\left(\frac{\phi}{2}\right)$$

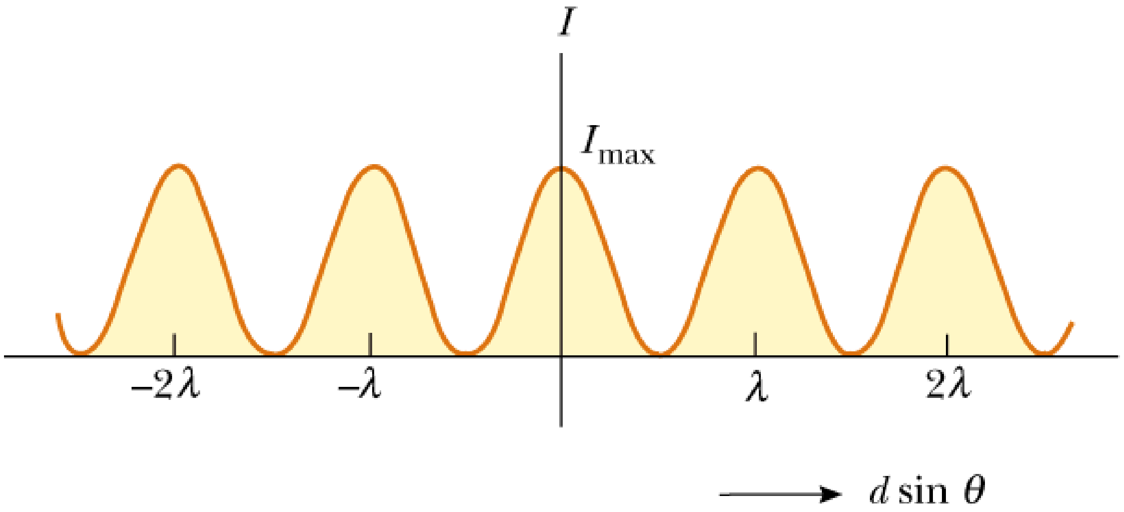
$$I = I_{\max} \cos^2\left(\frac{\pi d \sin \theta}{\lambda}\right)$$

$$\boxed{I = I_{\max} \cos^2\left(\frac{\pi d}{\lambda L} y\right)}$$

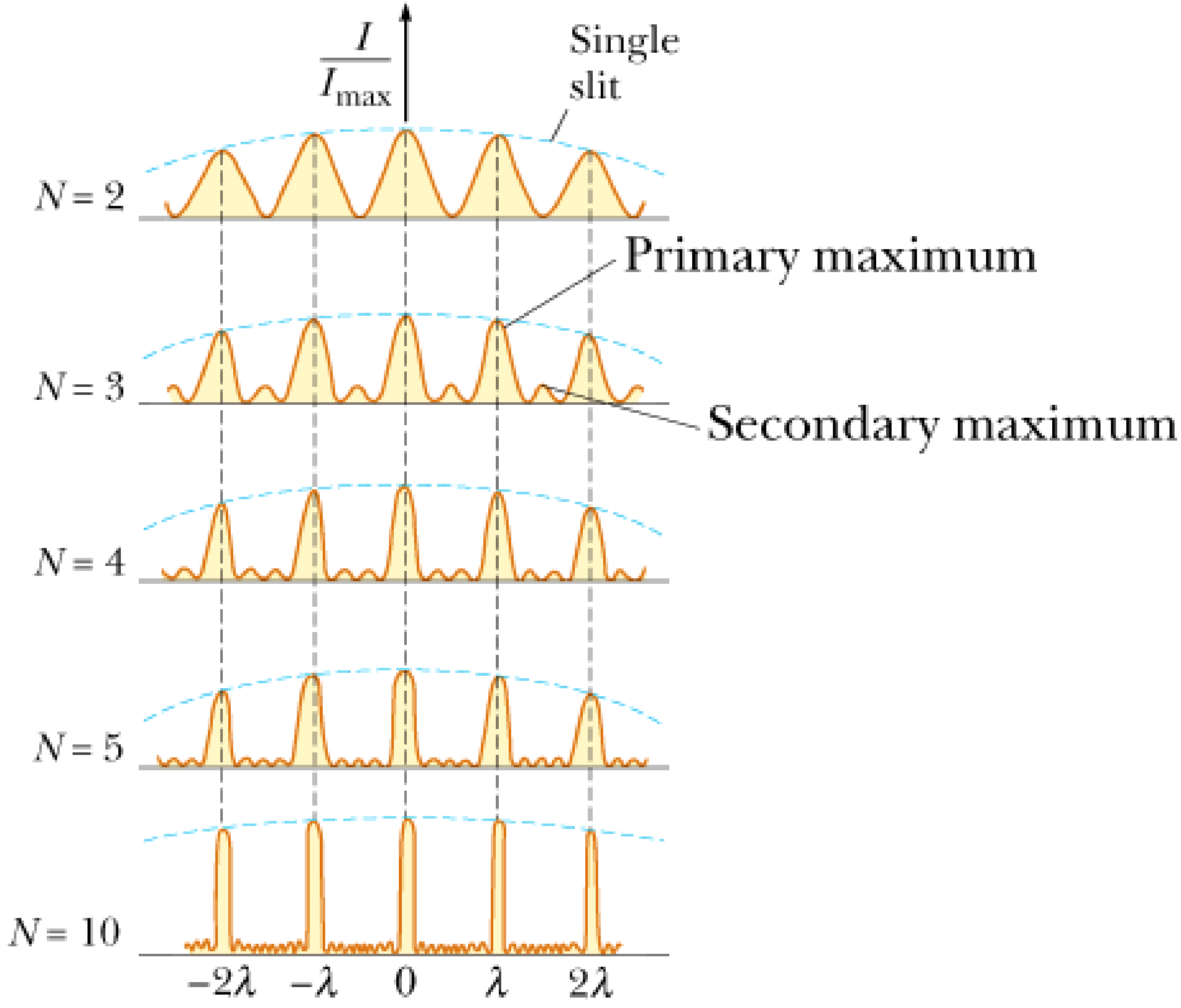
Intensity Distribution of the Interference Pattern

- Interference depends on the relative phase of the two waves.
- It also depends on the path difference between them.
- The resultant intensity at a point is proportional to the square of the resultant electric field at that point.

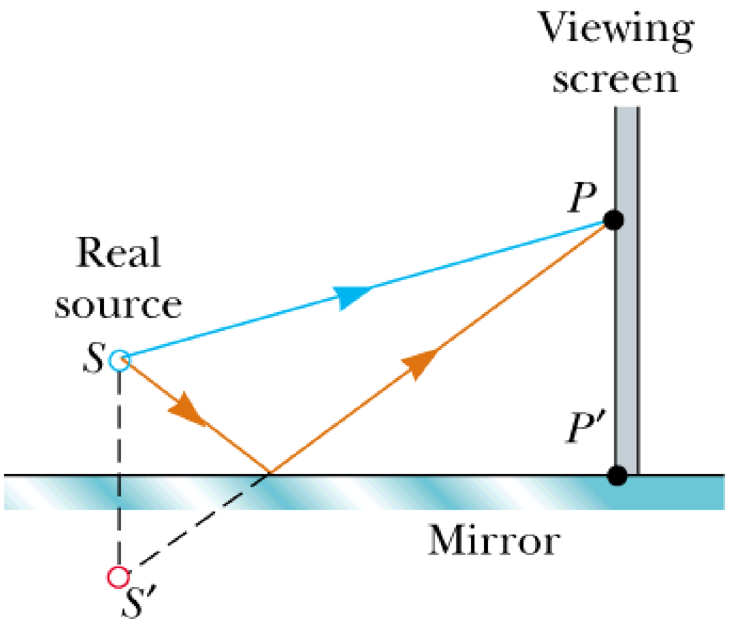
$$I \propto (E_1 + E_2)^2 \quad \textbf{not} \quad E_1^{\,2} + E_2^{\,2}$$



Multiple Slit Patterns



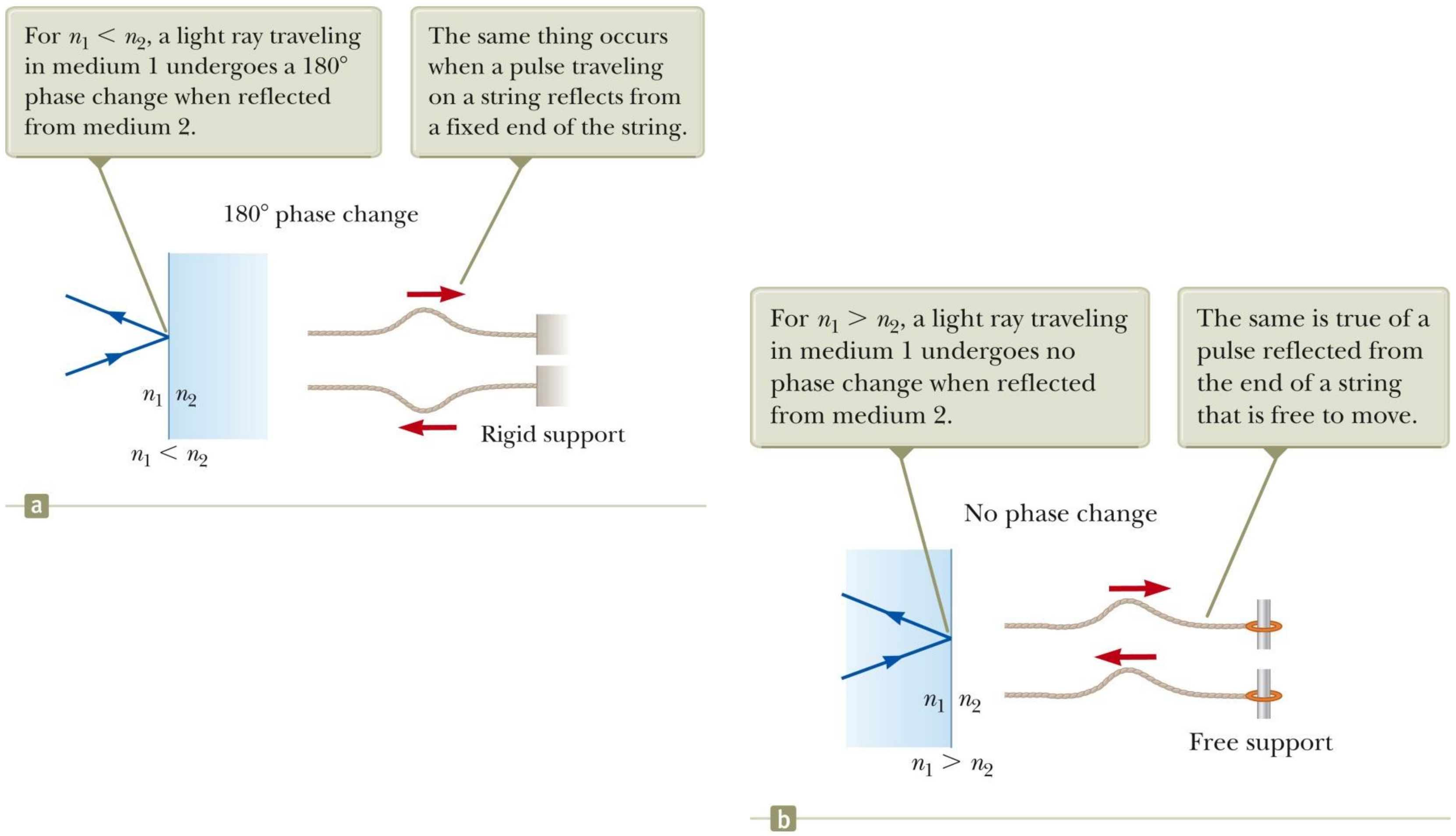
Change of Phase in Reflection



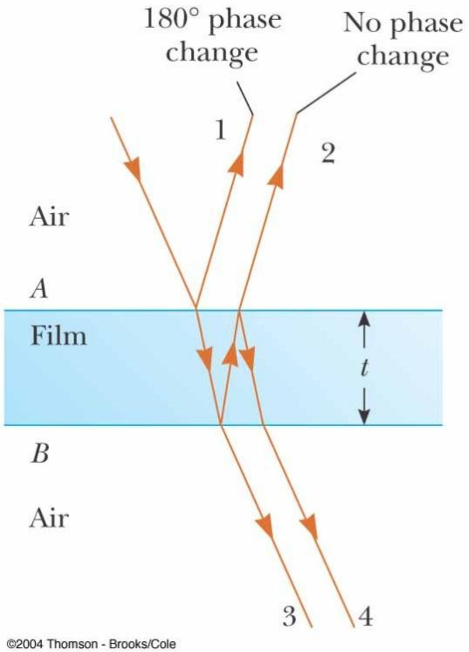
The positions of the fringes are reversed compared to Young's experiment

An EM wave undergoes a phase change of 180° upon reflection from a medium that has a higher index of refraction than the one in which it is traveling.

String Analogy



Interference in Thin Films



- A wave traveling from a medium of index of refraction of n_1 towards a medium with index of refraction of n_2 undergoes a 180° phase change upon reflection if $n_2 > n_1$ and no phase change if $n_2 < n_1$.
- The wavelength of light λ_n in a medium with index of refraction n is given by, $\lambda_n = \lambda / n$.

For constructive interference

$$2t = \left(m + \frac{1}{2}\right)\lambda_n \qquad 2nt = \left(m + \frac{1}{2}\right)\lambda \qquad m = 0,1,2,\ldots$$

For destructive interference

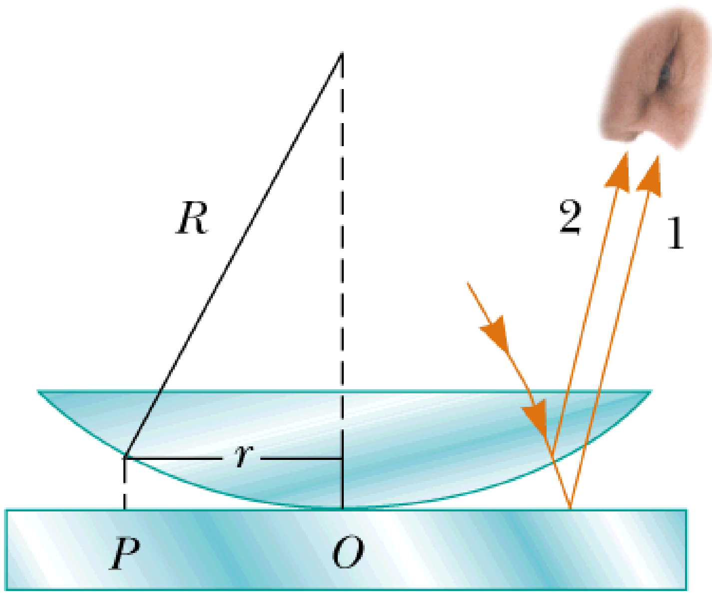
$$2nt = m\lambda \qquad m = 0,1,2,\ldots$$

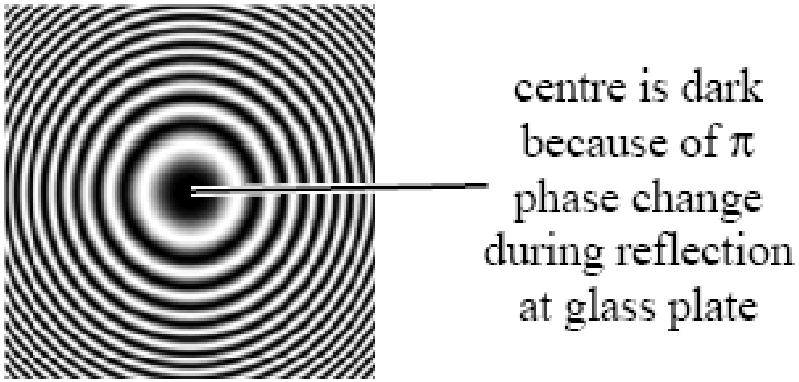
Concept Question

Two identical slides in air are illuminated with monochromatic light. The slides are exactly parallel, and the top slide is moving slowly upward. What do you see in top view?

1. all black
2. all bright
3. fringes moving apart
4. sequentially all black, then all bright
5. none of the above

Newton's Rings

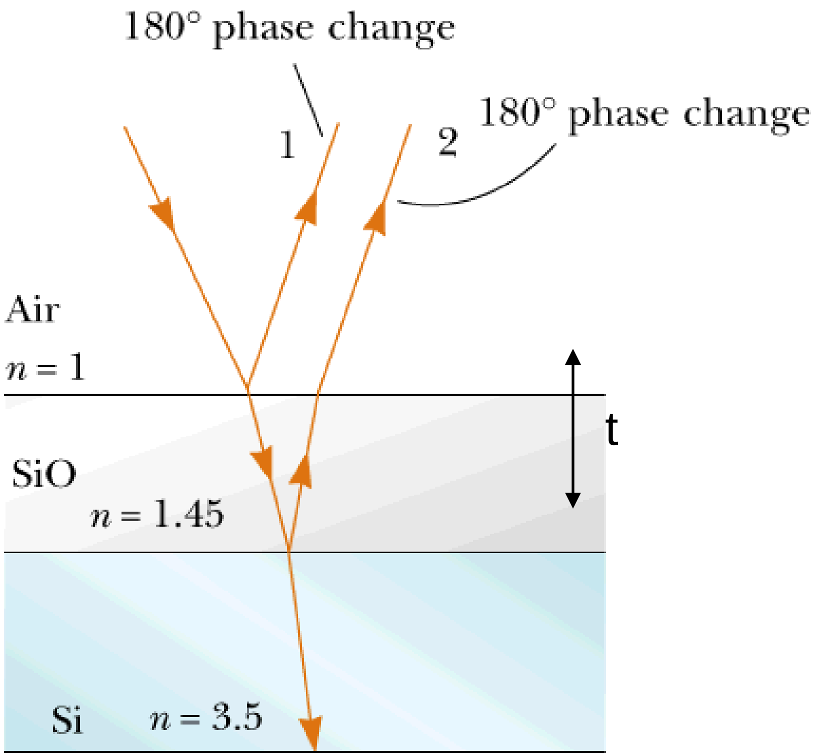




For destructive interference $r \approx \sqrt{m\lambda R / n}$

For constructive interference $r \approx \sqrt{(m+1/2)\lambda R / n}$

Non-reflective Coatings



Since both paths have the same phase change at the interfaces, take only the path differences into account.

$$2t_m = \left(\frac{1}{2} + m\right)\lambda_n$$

For destructive interference

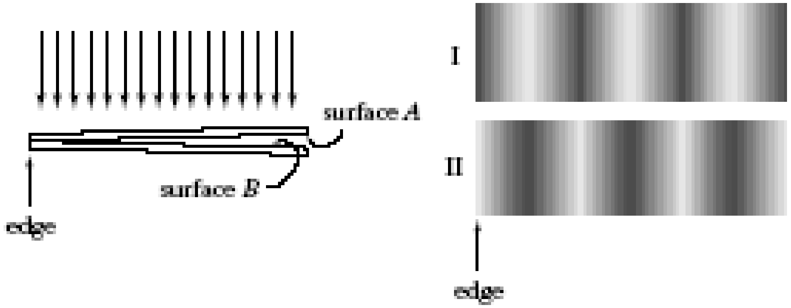
$$2t_m = \left(\frac{1}{2} + m\right)\frac{\lambda}{n}$$

Example: λ = 550 nm, no reflection →

$$t = \frac{\lambda}{4n} = \frac{550nm}{4(1.45)} = 94.8nm$$

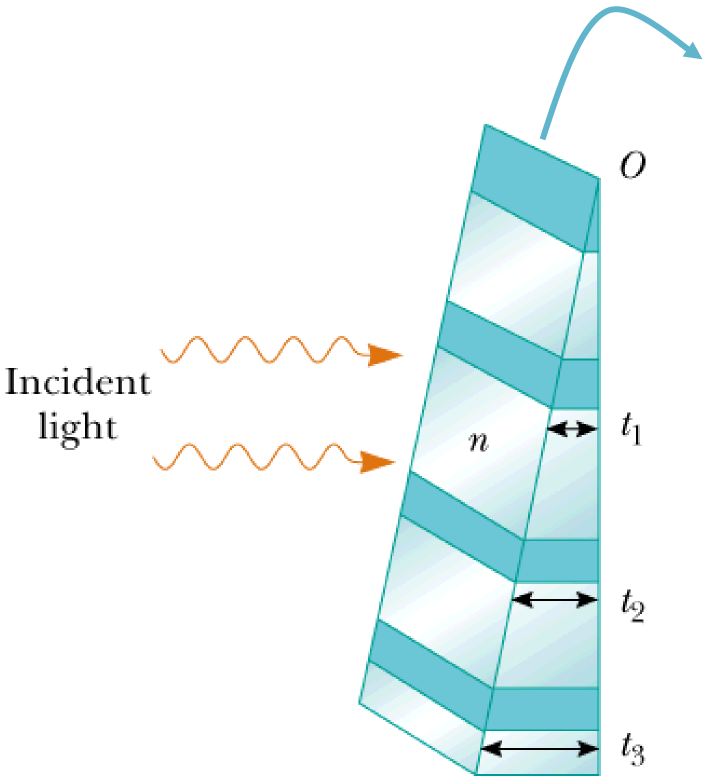
Concept Question

Monochromatic light shines on a pair of identical glass microscope slides that form a very narrow wedge. The top surface of the upper slide and the bottom surface of the lower slide have special coatings on them so that they reflect no light. The inner two surfaces (*A* and *B*) have nonzero reflectivities. A top view of the slides looks like



1. I.
2. II.

Interference in a Wedge Shaped Film

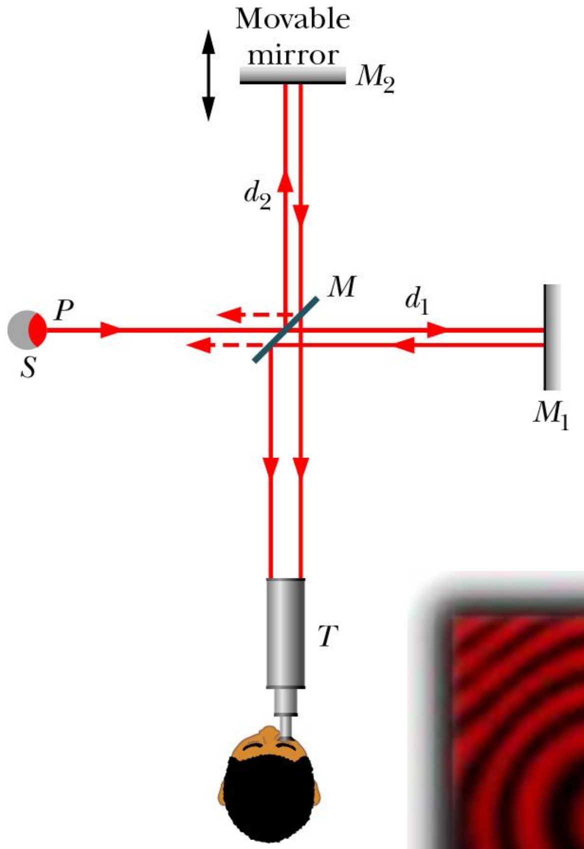


Destructive interference at the tip because of 180° phase change for the front surface and no phase change for the back surface.

$$2nt_m = m\lambda$$ For destructive interference

$$2nt_m = \left(\frac{1}{2} + m\right)\lambda$$ For constructive interference

Michelson Interferometer



- An interferometer can measure changes in length very accurately by observing the fringes.
- The phase difference is due to the path length difference between the two arms of the interferometer.

$$\delta = 2(d_1 - d_2)$$

- If a thin material is inserted in one arm, the change in the number of fringes is the change in the path difference.

Summary of Interference Conditions

- Interference depends on the phase difference between two waves.
  - If $\Delta\phi = (2m+1)\pi$ then we have destructive interference
  - If $\Delta\phi = 2m\pi$ then we have constructive interference
- This phase difference can have two sources:
  - Path length difference (if $\delta = \lambda$ then $\Delta\phi = 2\pi$)
  - Reflection

δ	Relative phase change on reflections	
	0	π
$m\lambda$	Const.	Dest.
$(m+½)\lambda$	Dest.	Const.

For Next Class

- Reading Assignment
  - Chapter 38 - Diffraction and Polarization
- WebAssign: Assignment 15